

1) PROPERTIES OF LOGARITHMS

- $\log_a mn = \log_a m + \log_a n$ ■ $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$
- $\log_a m^n = n \log_a m$ ■ $\log_a m = \frac{\log_b m}{\log_b a}$

Some Standard Derivatives

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| (i) $\frac{d}{dx} (\sin x) = \cos x$ | (ii) $\frac{d}{dx} (\cos x) = -\sin x$ |
| (iii) $\frac{d}{dx} (\tan x) = \sec^2 x$ | (iv) $\frac{d}{dx} (\sec x) = \sec x \tan x$ |
| (v) $\frac{d}{dx} (\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$ | (vi) $\frac{d}{dx} (\cot x) = -\operatorname{cosec}^2 x$ |
| (vii) $\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ | (viii) $\frac{d}{dx} (\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$ |
| (ix) $\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$ | (x) $\frac{d}{dx} (\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$ |
| (xi) $\frac{d}{dx} (\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$ | (xii) $\frac{d}{dx} (\cot^{-1} x) = \frac{-1}{1+x^2}$ |
| (xiii) $\frac{d}{dx} (x^n) = nx^{n-1}$ | (xiv) $\frac{d}{dx} (\text{constant}) = 0$ |
| (xv) $\frac{d}{dx} (e^x) = e^x$ | (xvi) $\frac{d}{dx} (\log_e x) = \frac{1}{x}, x > 0$ |
| (xvii) $\frac{d}{dx} (a^x) = a^x \log_e a, a > 0$ | |

2) LOGARITHMIC DIFFERENTIATION

If the given function is a $(function)^{function}$ or product or quotient of many functions we use logarithmic differentiation.

Let $y = f(x)^{\varphi(x)}$. Taking logarithm (with base e) on both sides

$$\log y = \log f(x)^{\varphi(x)}$$

$\log y = \varphi(x) \log f(x)$. Differentiating both sides w.r.to x

$$\frac{d(\log y)}{dx} = \frac{d(\varphi(x) \log f(x))}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} = \varphi(x) \frac{1}{f(x)} f'(x) + \log f(x) \varphi'(x)$$

$$\therefore \frac{dy}{dx} = y \left(\varphi(x) \frac{1}{f(x)} f'(x) + \log f(x) \varphi'(x) \right)$$

Here $f(x)$ and $\varphi(x)$ must always be positive as otherwise their logarithms are not defined